A Probability Ranking Principle for Interactive IR

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Outline

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- The classical PRP
- Questioning the PRP assumptions
- Interactive Retrieval

Approach
- Requirements for an IIR-PRP
- Basic Assumptions
- Abstraction: Situations with Lists of Choices

The Model
- Choices
- Selection lists
- Ranking of choices

Conclusion and Outlook
The classical PRP

- Task: Retrieve relevant documents
- Relevance of a document to a query is independent of other documents
- Scanning through the ranked list is the major task of the user (and the only one considered)
Questioning the PRP assumptions

- Relevance depends on documents the user has seen before
- Relevance judgment is not the most expensive task for a user
Interactive Retrieval

- User has a rich set of interaction possibilities
  - (re)formulate query
  - selection based on summaries of various granularity
  - select related terms from list
  - follow document link
  - relevance judgment

- Information need changes during a search
- No theoretic foundation for constructing IIR systems
Requirements for an IIR-PRP

- Consider the complete interaction process
- Allow for different costs for different activities
- Allow for changes of the information need
Example: Query Trails
Basic Assumptions

- Focus on a functional level of interaction (usability issues disregarded here)
- System presents list of choices to the user
- Users evaluate choices in linear order
- Only positive decisions/choices are of benefit for a user
Examples of decision lists

- ranked list of documents
- list of summaries
- list of document cluster
- KWIC list
- list of expansion terms
- links to related documents
- ...
Abstraction: Situations with Lists of Choices
Basic ideas

- A user moves from situation to situation
- In each situation $s_i$, the user is presented a list of (binary) choices $< c_{i1}, c_{i2}, \ldots, c_{in_i} >$
- The user decides about each of these choices sequentially
- The first positive decision moves the user to a new situation $s_j$
- A decision may be wrong, requiring backtracking
Probabilistic model focusing on single situation
Probabilistic Event space

$U_i$: Uses in situation $s_i$

$C_i$: choices in situation $s_i$

$J \subset U_i \times C_i$: judged choices

$A \subset J$: accepted choices

$R \subseteq A$: 'right' choices
Expected Benefit of a choice

\[ p_{ij} \text{ probability that the user will accept choice } c_{ij} \]

\[ q_{ij} \text{ probability that this decision was right} \]

\[ e_{ij} < 0: \text{ effort for evaluating the choice } c_{ij} \]

\[ b_{ij} > 0: \text{ resulting benefit from positive, correct decision} \]

\[ g_{ij} \leq 0: \text{ cost for correcting a wrong decision} \]

**Expected benefit of choice** \( c_{ij} \)

\[ E(c_{ij}) = e_{ij} + p_{ij} (q_{ij} b_{ij} + (1 - q_{ij}) g_{ij}) \]
Example

Web search: 'Java' → $n_0=290$ mio. hits

System proposes extension terms:

<table>
<thead>
<tr>
<th>term</th>
<th>$n_i$</th>
<th>$p_{ij}$</th>
<th>$b_{ij}$</th>
<th>$p_{ij}b_{ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>program</td>
<td>195 mio</td>
<td>0.67</td>
<td>0.4</td>
<td>0.268</td>
</tr>
<tr>
<td>blend</td>
<td>5 mio</td>
<td>0.02</td>
<td>4.0</td>
<td>0.08</td>
</tr>
<tr>
<td>island</td>
<td>2 mio</td>
<td>0.01</td>
<td>4.9</td>
<td>0.049</td>
</tr>
</tbody>
</table>

benefit $b_{ij} = \log \frac{n_0}{n_i}$
Strategies for maximizing expected benefit

\[ E(c_{ij}) = e_{ij} + p_{ij} (q_{ij} b_{ij} + (1 - q_{ij}) g_{ij}) \]
(assume that benefit \( b_{ij} \) and corr. effort \( g_{ij} \) are given)

1. minimize effort \( |e_{ij}| \) —
   but keep \( p_{ij} \) (selection prob.) and \( q_{ij} \) (success prob.) high

2. maximize \( p_{ij} \): user should choose \( c_{ij} \) whenever it is appropriate —
   but keep success probability \( q_{ij} \) high
   \( \Rightarrow \) increased effort \( e_{ij} \)

3. maximize \( q_{ij} \) by avoiding erroneous positive decisions
   \( \Rightarrow \) increased effort \( e_{ij} \)
Further remarks

\[ E(c_{ij}) = e_{ij} + p_{ij} \left( q_{ij} b_{ij} + (1 - q_{ij}) g_{ij} \right) \]

- Expected benefit should be positive. Choices with negative values should not be presented to a user.
- Methods for estimating parameters \( p_{ij}, q_{ij}, b_{ij}, e_{ij}, g_{ij} \): Issue of further research
- In the following, let \( a_{ij} = q_{ij} b_{ij} - (1 - q_{ij}) g_{ij} \) (“average benefit”)
  \[ E(c_{ij}) = e_{ij} + p_{ij} a_{ij} \]
Selection list

situation $s_i$ with list of choices $r_i = < c_{i1}, c_{i2}, \ldots, c_{i,n_i} >$

expected benefit of choice list:

\[
E(r_i) = e_{i1} + p_{i1} a_{i1} + \left(1 - p_{i1}\right) (e_{i2} + p_{i2} a_{i2} + \left(1 - p_{i2}\right) (e_{i3} + p_{i3} a_{i3} + \ldots \left(1 - p_{i,n-1}\right) (e_{in} + p_{in} a_{in}) )
\]

\[
= \sum_{j=1}^{n} \left(\prod_{k=1}^{j-1} (1 - p_{ik})\right) (e_{ij} + p_{ij} a_{ij})
\]
Expected benefit of a choice list

\[ E(r_i) = \sum_{j=1}^{n} \left( \prod_{k=1}^{j-1} (1 - p_{ik}) \right) (e_{ij} + p_{ij} a_{ij}) \]
Ranking of choices

Consider two subsequent choices $c_{il}$ and $c_{i,l+1}$

$$E(r_i) = \sum_{j=1}^{n} \left( \prod_{k=1}^{j-1} (1 - p_{ik}) \right) \left( e_{ij} + p_{ij} a_{ij} \right) + t_{i,l,l+1}$$

where

$$t_{i,l,l+1} = (e_{il} + p_{il} a_{il}) \prod_{k=1}^{l-1} (1 - p_{ik}) + (e_{i,l+1} + p_{i,l+1} a_{i,l+1}) \prod_{k=1}^{l} (1 - p_{ik})$$

analogously $t_{i,l+1,l}^{l+1}$ for $< \ldots, c_{i,l+1}, c_{il}, \ldots >$
Difference between alternative rankings

\[ d_{i,l,l+1} = \frac{t_{i,l+1} - t_{i,l+1,l}}{\prod_{k=1}^{l-1} (1 - p_{ik})} \]

\[ = e_{il} + p_{il}a_{il} + (1 - p_{il})(e_{i,l+1} + p_{i,l+1}a_{i,l+1}) -\]

\[ (e_{i,l+1} + p_{i,l+1}a_{i,l+1} + (1 - p_{i,l+1})(e_{il} + p_{il}a_{il})) \]

\[ = p_{i,l+1}(e_{il} + p_{il}a_{il}) - p_{il}(e_{i,l+1} + p_{i,l+1}a_{i,l+1}) \]

For \( d_{i,l,l+1} \geq 0 \), we get

\[ a_{il} - \frac{e_{il}}{p_{il}} \geq a_{i,l+1} - \frac{e_{i,l+1}}{p_{i,l+1}} \]
PRP for Interactive IR

\[ a_{il} - \frac{e_{il}}{p_{il}} \geq a_{i,l+1} - \frac{e_{i,l+1}}{p_{i,l+1}} \]

\[ \rightsquigarrow \text{Rank choices by decreasing values of} \]

\[ \varrho(c_{ij}) = a_{il} + \frac{e_{il}}{p_{il}} \]
Expected benefit: single choices vs. list

expected benefit:  \[ E(c_{ij}) = p_{ij} a_{ij} + e_{ij} \]

ranking criterion:  \[ \varrho(c_{ij}) = a_{il} + \frac{e_{il}}{p_{il}} \]

Example:

<table>
<thead>
<tr>
<th>choice</th>
<th>( p_{ij} )</th>
<th>( a_{ij} )</th>
<th>( e_{ij} )</th>
<th>( E(c_{ij}) )</th>
<th>( \varrho(c_{ij}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_1 )</td>
<td>0.5</td>
<td>10</td>
<td>-1</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>0.25</td>
<td>16</td>
<td>-1</td>
<td>3</td>
<td>12</td>
</tr>
</tbody>
</table>

\[ E(\langle c_1, c_2 \rangle) = 4 + 0.5 \cdot 3 = 5.5 \]

\[ E(\langle c_2, c_1 \rangle) = 3 + 0.75 \cdot 4 = 6 \]
IIR-PRP vs. PRP

\[ a_{il} + \frac{e_{il}}{p_{il}} \geq a_{i,l+1} + \frac{e_{i,l+1}}{p_{i,l+1}} \]

Let \( e_{ij} = -\bar{C} \), \( \bar{C} > 0 \) and \( a_{il} = C \):

\[ C - \frac{\bar{C}}{p_{il}} \geq C - \frac{\bar{C}}{p_{i,l+1}} \]
\[ \Rightarrow p_{il} \geq p_{i,l+1} \]

\( \rightsquigarrow \) Classic PRP still holds!
IIR-PRP: Observations

Rank choices by \( a_{ij} - \frac{e_{ij}}{p_{ij}} \)

- \( p_{ij} \) 'probability of relevance' still involved
- tradeoff between effort \( e_{ij} \) and benefit \( a_{ij} \)
- difference between PRP and IIR-PRP due to variable values for \( e_{ij} \) and \( a_{ij} \)
- IIR-PRP looks only for the first positive decision
Conclusion and Outlook

- Current IIR systems lack theoretic foundation
- Interactive IR as decision making
- User works on linear list of choices
- Positive choices move user to new situation, with (possibly) new choice list
- IIR-PRP is generalization of classical PRP
- Introduced new parameters
- Parameter estimation is issue of further research