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Exercises in Modeling Methods in Computer Science, Winter 2005/06
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Exercise 1  Submit until 31.10.2005, 4 pm

Mode of submission:

- Please submit the homework, on paper in the lowermost letterbox before the steel door of LF 135-139. „Modellierung“ is written on the letterbox.

- The dates for the submission of the homework are fixed and cannot be changed. Solutions that are submitted after 4 pm will not be considered and no points will be given.

- Solutions can be submitted in groups of up to 3 students. However each group can submit only a single common solution. Furthermore each member of the group should be in a position to explain the solution in the exercise.

- Please write the solution of each problem on a separate sheet of paper.

- Please write on each sheet of the solution
  - Names of all group members
  - The Matrikelnumbers of all group members
  - The number or the timings of the Übung in which the corrected homework should be returned

- Identical solutions or solutions that are very similar would be given 0 points.

- Please register yourself, if you haven’t done it already on the website of the lecture for the exercises.

- Please register yourself even if you don’t plan to attend the exercises, but would like to submit your homework.
Task 1: Additional semantical equivalencies

A set of atomic terms $A$, as well as the set of boolean terms $T_A$ that are built from $A$ are given. Prove that for every boolean term $T$ from $T_A$ the following is valid:

(a) $T \land \neg T \equiv 0$
(b) $T \lor \neg T \equiv 1$
(c) $0 \land T \equiv 0$
(d) $1 \land T \equiv T$

8 Points

Task 2: On a syntactical and semantical level

The set of atomic terms $A = \{A_1, A_2, A_3\}$ and the following boolean terms are given:

$T_1 := (A_1 \land A_2) \rightarrow A_3$
$T_2 := (A_1 \rightarrow A_2) \rightarrow (A_1 \rightarrow A_3)$
$T_3 := \neg (\neg (A_1 \rightarrow A_2) \lor (A_1 \rightarrow A_2)) \land A_3$

(a) Show the semantical equivalence of $T_1$ and $T_2$ over $A$

- on a syntactical level (transforming the terms like in the lecture notes, with adequate reasons for each and every step).
- on a semantic level (using the truth table)

(b) Show on a syntactic level, that there exists no truth assignment $\beta : A \rightarrow \{0, 1\}$ with $I_\beta(T_3) = 1$. Subsequently what can be said about $T_3$?

Hint:
In addition to the semantical equivalences in the lecture notes, you are also allowed to use the equivalences of problem 1 and the equivalences $1 \lor T \equiv 1$ and $0 \lor T \equiv T$.

12 Points

Additional problem (will not be corrected):

The set of atomic terms $A = \{R, N, S\}$, and the definitions in lesson 1 of the lecture notes are given.

(a) Is the following chain of symbols

$$(R \rightarrow N) \land (R \rightarrow \neg S) \land (S \rightarrow N) \rightarrow (R \leftrightarrow N)$$

a boolean term?

If yes, why? If no, what should be done to change the chain into a syntactically correct boolean term, without changing the chain of symbols at all?
(b) What does the above chain of symbols mean when the symbols and the operators have the following meaning:

- $R$ It’s raining.
- $N$ The road is wet.
- $S$ The road cleaners are put into service.
- $\leftrightarrow$ if and only if

(c) After correcting any discrepancies (if any) in part a), what titles should the columns of the truth table for the corresponding boolean term have? Why?