Exercise 6 Submit until 06.12.2005, 4 p.m.

These tasks are devoted to predicate logic of the first level. So, to feel prepared to it, read the corresponding paragraphs in the script one more time (pages 20-35), and make a list of definitions from it (do not submit, will not be corrected).

Task 11: Fundamentals

(a) Given the following formulas of predicate logic:

(i) $\exists x B(x) \land \exists x R(x)$

(ii) $\forall x \forall y (D(x, y) \rightarrow E(x, y))$

(iii) $\forall x (R(x) \rightarrow F(x))$

(iv) $\forall x \forall y (F(y) \land E(x, y) \rightarrow F(x))$

(v) $\forall x (B(x) \rightarrow \neg F(x))$

Please translate the formulas into sentences of natural language. Predicates should have the following meanings:

- $B(x)$ $x$ is a blue dragon
- $R(x)$ $x$ is a red dragon
- $D(x, y)$ $x$ devours $y$
- $E(x, y)$ $x$ is faster than $y$
- $F(x)$ $x$ can fly

(b) Formulate according to part a) next two propositions as formulas of predicate logic. Define, when needed, additional predicates or functional symbols.

(i) Blue dragons do not devour red dragons.

(ii) Every child of a blue dragon is a red dragon and every child of a red dragon is a blue dragon.

(c) Give a model for the set of formulas from part a). How many models does it have, and why?

5 + 2 + 3 = 10 Points
Task 12: Prenex form and skolem form

Take up the formulas from task 11. Suppose that, eventually, we want to prove, that from formulas of part a) follows that blue dragons do not devour red dragons (see part b) (i)). This needs some prior steps, which are to be done in this task 12.

(a) Form one overall formula, in order to show with a predicate logic resolution, that

\[
\{ \text{Formulas from part a)}\} \models \text{Formula from part b(i)}
\]

Hints:

- The overall formula should just be composed from formulas from part 11 (plus appropriate operators). It should not be further re-formed.
- Do not carry out the actual resolution, this task is about the preparational steps only.
- In Prolog, the formula from 11 b(i) would correspond to a query. Similarly, the formula from 11 a) would correspond to a Prolog program.

(b) Set the overall formula from part a) to an adjusted prenex form (deu. vereinigte Pränexform) (see script page 26 at the bottom). Specify all steps in between.

(c) Produce from the prenex normal form a skolem form.

(d) Give the KNF and the corresponding set of clauses.

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2 + 3 + 3 + 2 = 10 \text{ Points}
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