Task 13: Unification

Let $a$, $b$ and $c$ be constants, $f$ and $g$ be functions, $x, y, z, x'$ be variables, and $P$ and $Q$ be predicates.

Apply the unification algorithm on the following sets of expressions. If a set of expressions is unifiable, then give also the most general unifier.

(a) \{ $P(x, f(y)), P(g(y, a), f(b)), P(g(b, x'), z)$ \}

(b) \{ $P(f(x), f(f(a))), Q(f(x), f(f(a)), g(z, z, z))$ \}

(c) \{ $Q(a, x, g(a, b, c)), Q(a, f(z), g(a, b, y)), Q(a, f(f(x')), g(a, x, g(c, b, a)))$ \}

6 Points

Task 14: Ground resolution

Given a constant $c$ and a function with a single argument $f$. Let $x, y$ and $z$ be variables.

Show with predicate logic ground resolution, the unsatisfiability of the following formulas given in the Skolemform:

$$F = \forall x \forall y \forall z ((\neg P(f(c)) \lor \neg P(y) \lor Q(y)) \land P(f(z)) \land (\neg P(f(f(x))) \lor \neg Q(f(x))))$$

(a) Give the set of clauses.

(b) Show the resolution graphically, also showing the substitutions used.

(c) Which ground instances of the clauses must be created, in order to arrive at the empty clause?

6 Points
Task 15: Resolution in predicate logic

Given are constants \(a\), \(b\), \(c\) and \(d\). Let \(x_i\) and \(y_i\) be variables.

Prove that the following set of clauses is unsatisfiable (unrealizable) (see also problem 12, sheet 6) using the Robinson method for resolution in predicate logic:

\[
\begin{align*}
\{B(a)\}, \{R(b)\}, \{\neg D(x_3, y_3), E(x_3, y_3)\}, \{\neg R(x_4), F(x_4)\}, \\
\{\neg F(y_5), \neg E(x_5, y_5), F(x_5)\}, \{\neg B(x_6), \neg F(x_6)\}, \\
\{B(c)\}, \{R(d)\}, \{D(c, d)\}
\end{align*}
\]

(a) Show the resolution graphically along with the substitutions used.

(b) Which ground instances of clauses must be created to arrive at the empty clause?

5 Points

Task 16: Proving universal validity

Given any formula \(F\) in first-order predicate logic, how do we show the universal validity of \(F\) with resolution for predicate logic? Specify the necessary steps in between.

3 Points