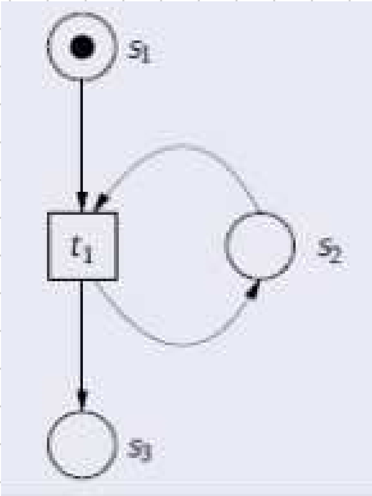
Modellierung 13.11.13

Notiztitel 13.11.2013

$$C = \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & -1 & -1 \end{pmatrix} \qquad \vec{u}_1 = \begin{array}{c} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \end{array}\begin{array}{l} t_1 \\ t_2 \\ t_3 \end{array} \qquad \vec{u}_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

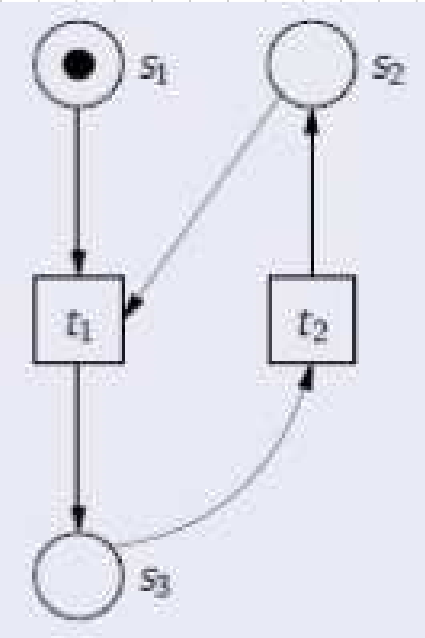
$$C \cdot \vec{u}_1 = \begin{pmatrix} -1 \cdot 1 + 1 \cdot 0 + 0 \cdot 0 \\ -1 \cdot 1 + 0 \cdot 0 + 1 \cdot 0 \\ 2 \cdot 1 - 1 \cdot 0 - 1 \cdot 0 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} \qquad C \cdot \vec{u}_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$\begin{array}{c|ccc} & t_1 & t_2 & t_3 \\ \hline 1 & -1 & 1 & 0 \\ 2 & -1 & 0 & 1 \\ 3 & 2 & -1 & -1 \end{array} \cdot \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}\begin{array}{l} t_1 \\ t_2 \\ t_3 \end{array} = \begin{pmatrix} -1 \cdot 2 + 1 \cdot 2 + 0 \cdot 1 \\ -1 \cdot 2 + 0 \cdot 2 + 1 \cdot 1 \\ 2 \cdot 2 - 1 \cdot 2 - 1 \cdot 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$



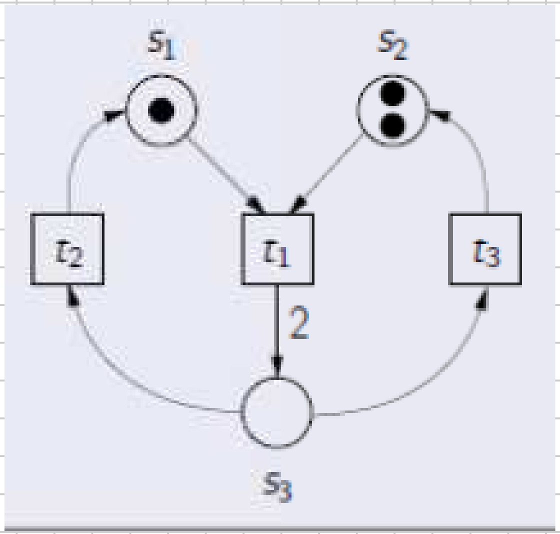
$$C = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \qquad \vec{u} = (1)$$

$$\underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}_{\vec{m}_0} + \underbrace{\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}}_{C} \cdot \underbrace{(1)}_{\vec{u}} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$



$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 & 0 \\ -1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1\cdot 1 + 0\cdot 1 \\ -1\cdot 1 + 1\cdot 1 \\ 1\cdot 1 - 1\cdot 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

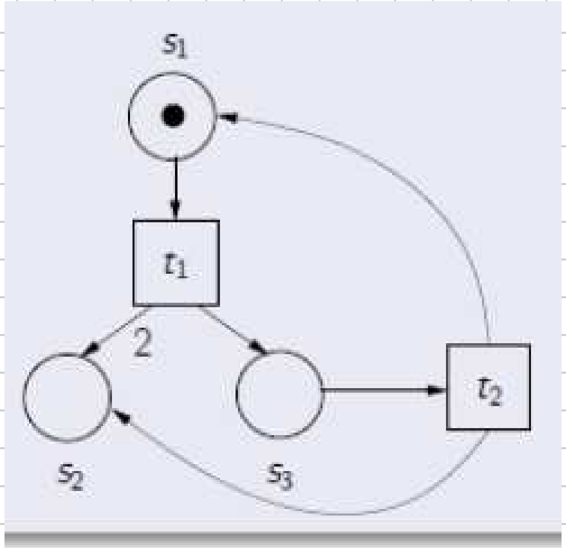


$$\underbrace{\begin{pmatrix}2\\2\\0\end{pmatrix}}_{\vec{m}} = \underbrace{\begin{pmatrix}1\\2\\0\end{pmatrix}}_{\vec{m}_0} + \underbrace{\begin{pmatrix}-1 & 1 & 0\\ -1 & 0 & 1\\ 2 & -1 & -1\end{pmatrix}}_{C} \cdot \underbrace{\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}}_{\vec{u}}$$

$$\begin{aligned}2 &= 1 - u_1 + u_2\\ 2 &= 2 - u_1 + u_3\\ 0 &= 0 + 2u_1 - u_2 - u_3\end{aligned}$$

$(1)+(2)+(3)$: $4 = 3$ ↯

Markierung $\begin{pmatrix}2\\2\\0\end{pmatrix}$ nicht erreichbar!



$$\begin{pmatrix}1\\0\\0\end{pmatrix} + \begin{array}{c|cc} & t_1 & t_2 \\ \hline s_1 & -1 & 1 \\ s_2 & 2 & 1 \\ s_3 & 1 & -1 \end{array} \cdot \begin{pmatrix}u_1\\u_2\end{pmatrix} = \begin{pmatrix}1\\20\\0\end{pmatrix}$$

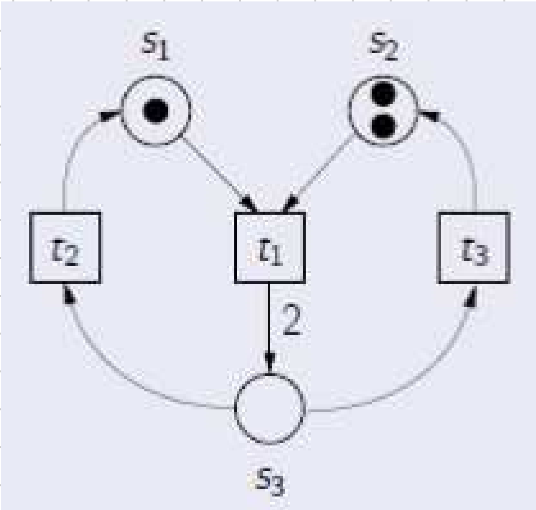
$$1 - u_1 + u_2 = 1$$

$$0 + 2u_1 + u_2 = 20 \quad | \quad 3u_1 = 20$$

$$0 + u_1 - u_2 = 0 \quad | \quad u_1 = u_2$$

$$u_1 = 6\tfrac{2}{3} \quad ↯$$

$$\begin{tikzcd}
 & (1,0,0) \arrow[dl, "t_1"'] & \\
(0,2,1) \arrow[r, "t_2"] & (1,\overset{\omega}{\cancel{2}},0) \arrow[d, bend right, "t_1"'] & \\
 & (0,\omega,1) \arrow[u, bend right, "t_2"'] &
\end{tikzcd}$$



$$\begin{array}{c|c} & \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ \begin{array}{c} s_1 \\ s_2 \\ s_3 \end{array} & \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & -1 & -1 \end{pmatrix} \end{array} \cdot \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-u_1 + u_2 = 0 \quad | \; u_1 = u_2$$
$$-u_1 + u_3 = 0 \quad | \; u_1 = u_3 = u_2$$
$$2u_1 - u_2 - u_3 = 0$$

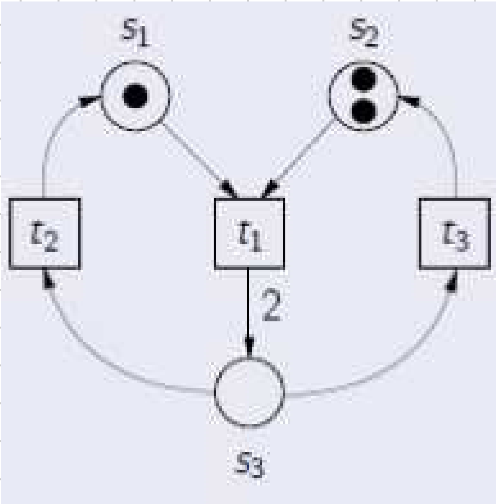
$$\vec{u} = \xi \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \xi \\ \xi \\ \xi \end{pmatrix}$$

$\xi \in \mathbb{N}_0$

$$\begin{tikzcd} (1,2,0) \arrow[d, "t_1"] & \\ (0,1,2) \arrow[r, "t_3"] \arrow[d, "t_2"] & (0,2,1) \arrow[ul, bend right, "t_2"'] \\ (1,1,1) \arrow[uu, bend left, "t_3"] & \end{tikzcd}$$

t_1, t_2, t_3

t_1, t_3, t_2



$$(\vartheta_1, \vartheta_2, \vartheta_3) \cdot \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & -1 & -1 \end{pmatrix} = (0 \quad 0 \quad 0)$$

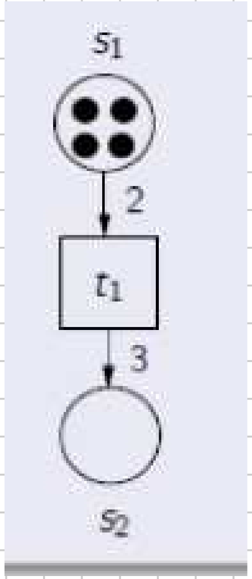
$$-\vartheta_1 - \vartheta_2 + 2\vartheta_3 = 0$$

$$\vartheta_1 \qquad - \vartheta_3 = 0 \mid \vartheta_1 = \vartheta_3$$

$$\vartheta_2 - \vartheta_3 = 0 \mid \vartheta_2 = \vartheta_3$$

$$\vartheta = (\lambda \quad \lambda \quad \lambda) = \lambda \cdot (1 \quad 1 \quad 1)$$

$$\vec{m} = \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} \qquad \vartheta \cdot \vec{m} = m_1 + m_2 + m_3 = 3 = \vartheta \cdot \vec{m}_0$$



$$(v_1 \;\; v_2) \cdot \begin{pmatrix} -2 \\ 3 \end{pmatrix} = (0)$$

$$-2v_1 + 3v_2 = 0$$

$$v_1 = \frac{3}{2} v_2$$

$$v = v_2 \left(\tfrac{3}{2} \;\; 1\right)$$

$$v = \ell \cdot (3 \;\; 2)$$

$$v \cdot \vec{m} = 3m_1 + 2m_2 = 12 = v \cdot \vec{m}_0$$

$$\vec{m} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} \qquad \vec{m}_2 = \begin{pmatrix} 0 \\ 6 \end{pmatrix} \qquad v \cdot \vec{m}_2 = 3 \cdot 0 + 2 \cdot 6 = 12$$

$$v \cdot \vec{m} = 3 \cdot 0 + 2 \cdot 5 = 10 \neq 12 \quad ↯$$