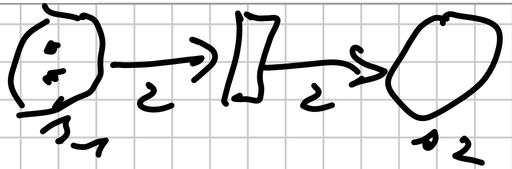
Modellierung 19.11.14

Notiztitel 19.11.2014



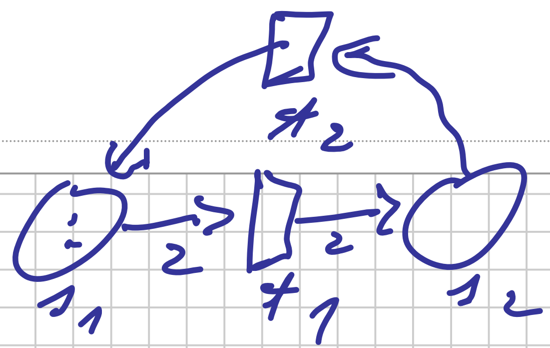
$$\binom{1}{1} = \binom{2}{0} + \binom{-2}{2} \cdot (\mu_1)$$

$$1 = 2 - 2\mu_1$$

$$1 = 0 + 2\mu_1$$

$$\mu_1 = \frac{1}{2} \ \text{↯}$$

nur ganzzahlige Lösungen zulässig $\mathbb{N}_0$



$$\binom{1}{1} = \binom{2}{0} + \begin{pmatrix} -2 & 1 \\ 2 & -1 \end{pmatrix} \cdot \binom{\mu_1}{\mu_2}$$

$$-1 = -2\mu_1 + \mu_2$$

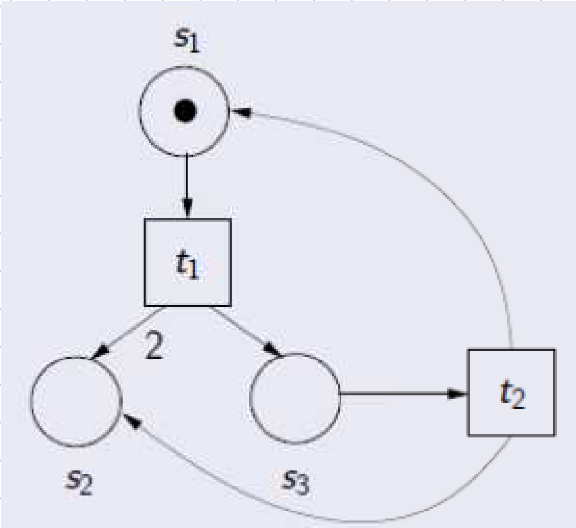
$$1 = 2\mu_1 - \mu_2$$

$$\mu_2 = 2\mu_1 - 1$$

$$\mu_1 = i \in \mathbb{N}$$

$$\mu_2 = 2i - 1$$

$$\binom{1}{1}, \binom{2}{3}, \quad \cancel{\binom{1/2}{0}}$$



$$\begin{pmatrix}1\\20\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix} + \begin{pmatrix}-1 & 1\\2 & 1\\1 & -1\end{pmatrix} \cdot \begin{pmatrix}u_1\\u_2\end{pmatrix}$$

$$0 = -u_1 + u_2$$

$$20 = 2u_1 + u_2 \quad | \; 3u_1 = 20$$

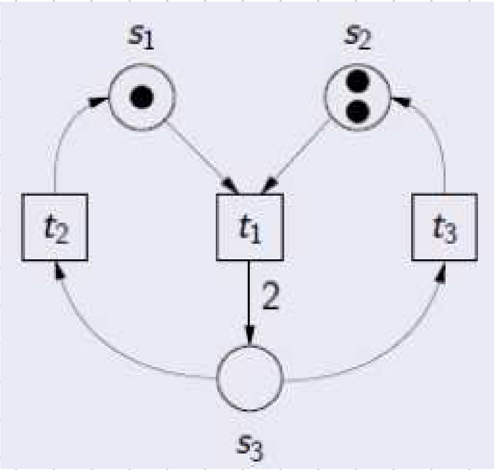
$$0 = u_1 - u_2 \quad | \; u_1 = u_2$$

$$u_1 = 6\tfrac{2}{3} \; ϟ$$

$$(1,0,0) \xrightarrow{t_1} (0,2,1) \xrightarrow{t_2} (1,\overset{\omega}{\cancel{3}},0)$$

$$(1,\omega,0) \underset{t_2}{\overset{t_1}{\rightleftarrows}} (0,\omega,1)$$

(1,20,0) wäre laut Ü-Graph möglich



$$
\begin{array}{c|ccc}
 & t_1 & t_2 & t_3 \\
s_1 & & & \\
s_2 & & & \\
s_3 & & &
\end{array}
\quad
\begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & -1 & -1 \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}
$$

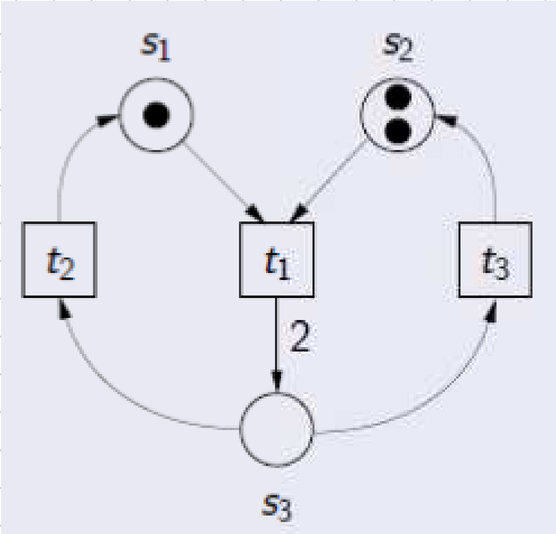
$$
\begin{aligned}
-u_1 + u_2 \qquad &= 0 \quad | \; u_1 = u_2 \\
-u_1 \qquad + u_3 &= 0 \quad | \; u_1 = u_3 \\
2u_1 - u_2 - u_3 &= 0
\end{aligned}
$$

$$\vec{u} = k \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \; k \in \mathbb{N}_0$$

$$
\begin{tikzcd}
(1,2,0) \arrow[d, "t_1"] & \\
(0,1,2) \arrow[r, "t_3"] \arrow[d, "t_2"] & (0,2,1) \arrow[ul, "t_2"'] \\
(1,1,1) \arrow[uu, bend left, "t_3"] &
\end{tikzcd}
$$

$m_0 \, [t_1, t_2, t_3 \rangle \, m_0$

$m_0 \, [t_1, t_3, t_2 \rangle \, m_0$



$$(v_1, v_2, v_3) \cdot \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & -1 & -1 \end{pmatrix} = (0\ 0\ 0)$$

$$-v_1 - v_2 + 2v_3 = 0$$

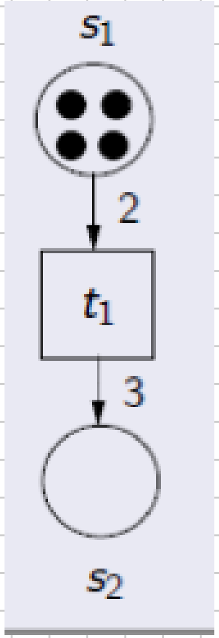
$$v_1 - v_3 = 0 \quad \Big| \quad v_1 = v_3$$

$$v_2 - v_3 = 0 \quad \Big| \quad v_2 = v_3$$

$$v = (\xi\ \ \xi\ \ \xi) = \xi \cdot (1\ 1\ 1) \quad \xi \in \mathbb{N}_0$$

$$\vec{m} = \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} \qquad v \cdot \vec{m} = m_1 + m_2 + m_3 = 3 = v \cdot \vec{m}_0$$

$\vec{m} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$ erreichbar? nein, da $m_1 + m_2 + m_3 = 4 \neq 3$



$$(v_1 \quad v_2) \cdot \begin{pmatrix} -2 \\ 3 \end{pmatrix} = (0)$$

$$-2v_1 + 3v_2 = 0$$

$$v_1 = \frac{3}{2} v_2$$

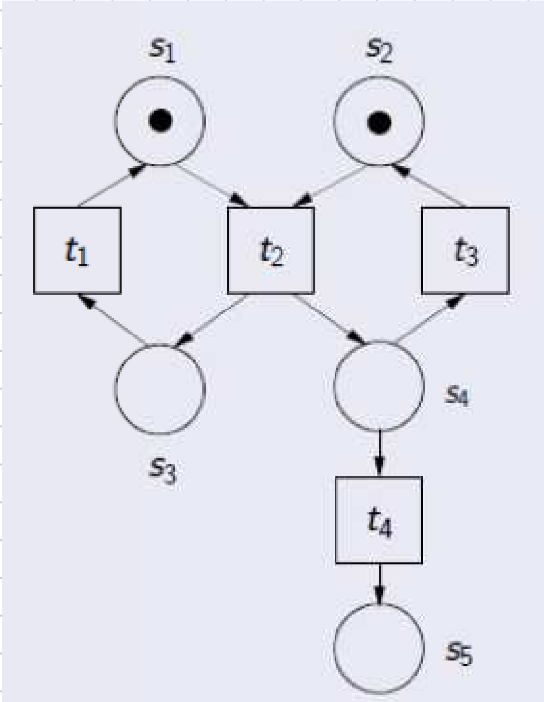
$$v = v_2 \left(\frac{3}{2} \quad 1 \right)$$

$$v = \ell \cdot (3 \quad 2)$$

$$v \cdot \vec{m} = 3m_1 + 2m_2 = 12 = v \cdot \vec{m_0}$$

$$\vec{m} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} ? \quad v \cdot \vec{m} = 3 \cdot 0 + 2 \cdot 5 = 10 \neq 12$$

$$m_2 = \begin{pmatrix} 0 \\ 6 \end{pmatrix} ? \quad v \cdot \vec{m} = 3 \cdot 0 + 2 \cdot 6 = \underline{\underline{12}} \checkmark$$



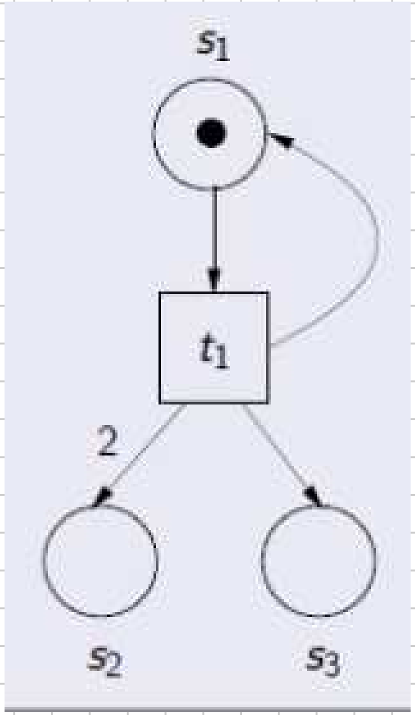
$$v \cdot \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} = (0\ 0\ 0\ 0)$$

$$\begin{aligned} v_1 \quad\quad - v_3 \quad\quad &= 0 \quad | \; v_1 = v_3 \\ -v_1 - v_2 + v_3 + v_4 &= 0 \\ v_2 \quad\quad - v_4 &= 0 \quad | \; v_4 = v_2 \\ -v_4 + v_5 &= 0 \quad | \; v_4 = v_5 \end{aligned}$$

$v = (a\ b\ a\ b\ b) = a \cdot (1\,0\,1\,0\,0) + b \cdot (0\,1\,0\,1\,1)$

$v \cdot \vec{m} = (a\ b\ a\ b\ b) \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \underline{a + 2b} \;\Big|\; v \cdot \vec{m}_0 = (a\ b\ a\ b\ b) \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \underline{a + b}$

$\neq$



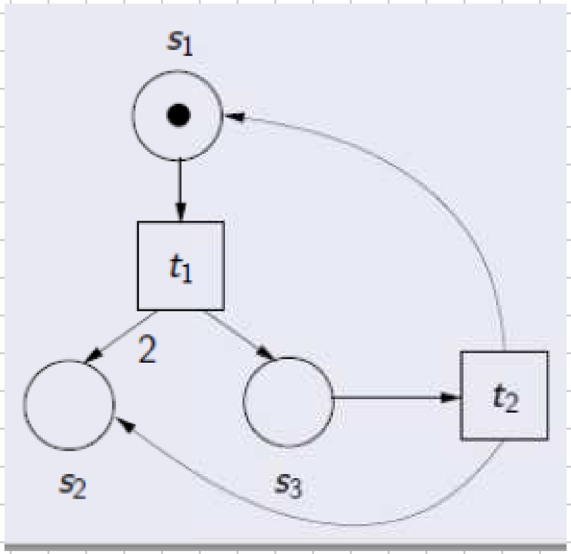
$$(v_1\ v_2\ v_3)\cdot\begin{pmatrix}0\\2\\1\end{pmatrix} = 0 \Rightarrow 2v_2 + v_3 = 0$$

$$v_3 = -2v_2$$

$$v = e\,(0\ \ 1\ \ -2)$$

$$v\cdot\vec{m} = 0\cdot m_1 + \underline{m_2 - 2m_3 = 0} = v\cdot\vec{m}_0$$

$$m_2 = 2m_3$$



$$(v_1\ v_2\ v_3) \cdot \begin{pmatrix} -1 & 1 \\ 2 & 1 \\ 1 & -1 \end{pmatrix} = (0\ 0)$$

$$-v_1 + 2v_2 + v_3 = 0$$
$$v_1 + v_2 - v_3 = 0$$

$$v_2 = 0$$
$$v_1 = v_3$$

$$v = \ell \cdot (1\ 0\ 1)$$

$$v \cdot \vec{m}_0 = (1\ 0\ 1) \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 1$$

$$\shortparallel$$

$$v \cdot \vec{m} = (1\ 0\ 1) \cdot \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = 1$$

gemäß S-Invariante möglich, tatsächlich nicht erreichbar